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Propagation of cosmic rays of energy 1–100 GeV in the Galaxy

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Abstract. In a recent paper we drew attention to the likelihood of the intensity of cosmic ray positrons being lower than expected on the basis of the simply 'leaky box' model of particle containment. Several possible explanations arise. Analysis of an essentially one-dimensional propagation, such as would arise from transport preferentially along spiral arms, shows that if a cosmic ray gradient exists—as is required for the most likely positron explanation—then parameters can be chosen which give the measured variation of mean path length for nuclei against energy in a natural way. This variation, near constancy below about 3 GeV/nucleon and a falling value above, follows from a diffusion coefficient and escape lifetime which are power laws of energy over the whole energy range.

In the present paper it is pointed out that in order to explain the data on cosmic ray positrons it is necessary to assume that the local cosmic rays are collected from a volume comparable with the size of the Galaxy. A specific model is proposed which is in agreement with existing experimental data on cosmic rays in the energy interval considered: 1-100 GeV.

1. Introduction

The manner in which cosmic rays propagate in the Galaxy is unclear. Some workers adhere to the idea of 'convection' along magnetic field lines and other are attracted to the hypothesis of a more conventional diffusion process. The interpretation of various cosmic ray properties obviously depends not only on propagation characteristics but also on the origin of the particles. Here we assume that the bulk are of Galactic origin; Ginzburg and Syrovatsky (1964) and others have drawn attention to the likelihood of supernovae as sources and Dodds *et al* (1975) have analysed γ -ray data to suggest the existence of cosmic ray gradients and thus, presumably, Galactic origin.

The analysis of cosmic ray positron results also appears to require cosmic ray gradients. We showed earlier (Giler *et al* 1977, to be referred to as I) that the positron flux appears to be smaller than expected on the basis of the conventional 'leaky box' model and this requires cosmic ray gradients in the Galaxy and spectral shapes which depend on position in the Galaxy. (The discrepancy had also been noticed by Dilworth *et al* (1974).) A number of alternative explanations arise including energy-dependent diffusion (and/or lifetime; Giler *et al* 1978, to be referred to as II) and production spectra which are themselves functions of position in the Galaxy.

It is generally assumed that, in the case of Galactic origin, low energy cosmic rays are of relatively local origin (e.g. Ginzburg and Ptuskin 1975). There are several arguments supporting this point of view. In particular, if we assume that the mean free path of cosmic rays is of the order of 10 pc (a commonly adopted value) and the mean age is of the order of 3×10^6 years, the distance to the source should be of the order of one kiloparsec.

This point of view is probably the main reason why, in most considerations, either the leaky box (homogeneous) or the local diffusion model is used. Both these models, however, give clear contradiction with the result obtained concerning the positron intensity, an understandable situation since the positron data require a significant gradient of cosmic rays. On the other hand, we know from anisotropy measurements that the gradient of cosmic rays on the scale of one kiloparsec is very small, so in order to explain the positron intensity it is necessary to assume that cosmic rays are collected from a much larger volume. To the question of an apparent contradiction with the expected mean free path (i.e. how a short mean free path can be reconciled with a large collecting volume) we will return later.

In the case of energy-dependent diffusion it was necessary to assume a diffusion coefficient D(E) which varied with proton energy (E) in a rather complex way in order to explain the variation of mean path length for nuclei $\lambda_N(E)$. Figure 1 shows the summarised values of $\lambda_N(E)$ from II and the best fit curve and figure 2 shows the required D(E) for the situations T = constant and $T \propto D^{-1/2}$. (In fact, we should deal with rigidity, R, rather than energy—this is done later. The distinction is unimportant.)

The manner in which D(E) and T(E) vary is, unfortunately, not known *a priori*. On a diffusion model the particle scattering arises from encounters with spatially varying magnetic field regions, such as would be present in 'clouds' of magnetised gas. Pulsar scintillation studies show the existence of strong fluctuations in the interstellar medium (ISM) on a scale of 10^{11} cm (i.e. $\sim 3 \times 10^{-8}$ pc) but there is no evidence for or



Figure 1. Mean path length for nuclei, λ_N , and the apparent grammage for positrons, λ_{e^+} , from I. The experimental values for λ_N relate to interpretations of the isotopic composition. The predicted dependence of λ_{e^+} on energy is shown with one standard deviation limit and a mean lifetime of $T = 3 \times 10^6$ yr.



Figure 2. Rigidity dependence of the diffusion coefficient used in II and the rigidity dependence adopted here.

against fluctuations on greater scales until one reaches some tens of parsecs where studies of the rotation of the polarised radiation from pulsars and extragalactic radio sources and of stellar polarisation have indicated scales of tens to hundreds of parsecs (Jokipii and Lerche 1969, Osborne *et al* 1973, and others). At the largest scale, Peters (1961) showed that the change of slope of the primary spectrum at $E_c \sim$ 5×10^{15} eV could be accounted for in terms of magnetised clouds having characteristic dimensions of several parsecs and magnetic fields of several microgauss so that D(E)was essentially independent of energy below E_c and increased rapidly above it. For clouds of constant diameter, $D(E) \propto E^2$, for $E > E_c$, giving a change of slope of the primary spectrum of $\Delta \gamma = 2$ (assuming primaries of one charge only), in contrast to the observed $\Delta \gamma \approx 0.6$ (the spectrum is represented by $j(E)dE = AE^{-\gamma} dE$). Peters (1961) pointed out that agreement could be restored by having a range of primary masses such that successive nuclei have break-points at different energies and the summed intensities give the required form. The primary composition is not known at the energies in question so that the validity of this argument cannot be checked.

An alternative argument has been put forward by Bell *et al* (1974). These authors drew attention to the fact that the clouds in the ISM have a range of sizes (see the work of Heiles 1967, Ames and Heiles 1970, and others) and if allowance is made for this, $D(E) \propto E$ above E_c and the change of slope can be made to be nearer to observation for a single primary mass. The scale of irregularities seen here makes us think that the hierarchy may exist right down to the scale of distance (~10⁻⁶ pc) of relevance to particles below 10¹¹ eV so that a power law representation for D(E) may be valid over an extended range.

Concerning particle lifetime the only comparatively direct measurements (from determination of the survival of ¹⁰Be) relate to energies below about 1 GeV/nucleon and they are currently discordant. Furthermore, they relate to the mean lifetime of the detected particles, \bar{i} , as distinct from the mean lifetime of the particles themselves from generation to escape from the Galaxy, T. For particles produced at a point

distance x from the sun the two are related by the expression given in II:

$$\bar{t} = \frac{T}{2} \left(\frac{|\mathbf{x}|}{(DT)^{1/2}} + 1 \right)$$
(1.1)

in the one-dimensional approximation so that the quantities are interrelated and one cannot disentangle T(E) from D(E) uniquely. However, from physical arguments it might be expected that if a power law fits D(E) then one should also fit T(E); this is the assumption made here.

The method used in II was to take a single source of particles and combinations of D(E) and T(E) which were not simple power laws but which enabled an explanation of the results on positrons after an exact fit had been made to the mean path length of nuclei, $\lambda_N(E)$. Here we adopt simple power law fits for D(E) and T(E) which, for reasons of propagation characteristics, fit $\lambda_N(E)$ and see whether they will also enable the positron results to be understood.

2. Diffusion of cosmic ray nuclei

The model follows that of II, i.e. one-dimensional diffusion with 'decay-like' escape. The dimension is identified with the spiral arm and the sources are taken to fall exponentially from the Galactic centre. The model is, of course, very simplified but it may have some merit insofar as spiral-arm aligned magnetic fields probably inhibit diffusion perpendicular to the arms; furthermore, the density of stars and thus perhaps many sources falls roughly exponentially with Galactocentric radius (Perek 1962, and others) and thus roughly with distance along the arms.

If x is the distance along the arm from the Galactic centre the source output is taken as

$$P(E, x) = q(E) \exp(-|x|/A)$$
 (2.1)

where q(E) is the source production spectrum assumed independent of x (a later paper will deal with the situation where sources of different types exist—flare stars, novae etc—and this condition is not satisfied).

The mean lifetime T(E) is identified with escape from the spiral arm and out of the Galaxy. Again we assume that this is independent of x, a poor assumption but, in view of the fact that D(E) is also taken to be similarly independent of x, any reasonable variation with x of both would probably only correspond to a scale change of A.

Using the basic expressions of II for diffusion from single sources the flux of nuclei at distance x from a source is

$$n(x) = \frac{q(E)}{2} \left(\frac{T}{D}\right)^{1/2} \exp\left(-\frac{|x|}{(DT)^{1/2}}\right)$$

the flux at distance x from the Galactic centre in the present situation is:

$$n(x) = \frac{qA(T/D)^{1/2}}{(1-A^2/DT)} \left[\exp\left(\frac{-|x|}{(DT)^{1/2}}\right) - \frac{A}{(DT)^{1/2}} \exp\left(-\frac{|x|}{A}\right) \right].$$
(2.2)

It is to be borne in mind that q, T and D are functions of E.

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The mean lifetime can be derived by integrating expressions of the form (1.1) to give

$$\bar{t} = \frac{1}{2}Tf((DT)^{1/2}, A, x)$$

where

$$f(\ldots) - 1 = \frac{1}{(DT)^{1/2}} \left[\frac{A(DT + A^2)}{DT - A^2} \exp\left(-\frac{|x|}{A}\right) + \left(x - \frac{2A^2(DT)^{1/2}}{DT - A^2}\right) \exp\left(-\frac{|x|}{(DT)^{1/2}}\right) \right] \\ \times \left[\exp\left(-\frac{|x|}{(DT)^{1/2}}\right) - \frac{A}{(DT)^{1/2}} \exp\left(-\frac{|x|}{A}\right) \right]^{-1}.$$
(2.3)

Figure 3 shows the f factor expressed as a function of $(DT)^{1/2}$ with A as parameter for x = 50 kpc (this value roughly fits Galactic properties).



Figure 3. Lifetime factor, *f*, as a function of $(DT)^{1/2}$ with *A* as parameter for the situation where x = 50 kpc (the value to be adopted here) and $\bar{t} = \frac{1}{2}Tf((DT)^{1/2})$.

The shape is interesting. For $(DT)^{1/2}/A \ll 1, f(\ldots) \rightarrow 2$ and thus $\overline{i} \rightarrow T$. Furthermore n(x) = P(E, x)T. This is the situation when the particles diffuse only a small distance compared with A. Thus the nearest sources contribute most and the mean life of the detected particles is the same as their inherent mean life, T.

For $(DT)^{1/2}/A \gg x/A$ and $(DT)^{1/2}/A \gg 1$ the expressions give $f(\ldots) \rightarrow 1$, i.e. $\overline{t} \rightarrow T/2$, and $n(x) = qTA/(DT)^{1/2}$.

Here, there is considerable diffusion and many of the detected particles have come from small x, in a short time.

When $DT = A^2$,

$$f(\ldots) \to \frac{1}{2} \left(\frac{x^2/DT}{[1+|x|/(DT)^{1/2}]} + 3 \right) \quad \text{and} \quad \bar{t} \to \frac{T}{4} \left(\frac{x^2/DT}{[x|/[1+(DT)^{1/2}]} + 3 \right).$$
(2.4)

Thus when $DT = A^2$ the flux is

$$n(x) = \frac{T}{2} \left(1 + \frac{|x|}{A} \right) q \exp\left(-\frac{|x|}{A} \right), \qquad (2.5)$$

that is,

$$n(x) = \frac{T}{2} \left(1 + \frac{|x|}{A} \right) P(x).$$
(2.6)

The increased mean life of detected particles arises because there are equal contributions from all values of x back to zero and, if $x \gg A$, the particles from small x moving slowly $(x \gg (DT)^{1/2})$ bring with them long lifetimes.

Inspection of f(...) against $(DT)^{1/2}/A$ and of figure 1 (λ_N against E) shows that suitable choice of parameters will enable a fit between them for the position of the solar system, i.e. by taking DT to grow with energy and x/A to give a maximum in f(...) one can have λ_N constant at low energy and falling with E at high energy.

3. Choice of parameters for D(R) and T(R) to fit $\lambda(R)$

Inspection of the stellar density as a function of Galactocentric radius indicates a value for A of the order of 10 kpc.

The situation corresponds, therefore, to the case where A = 10 in figure 3. A variety of possibilities arise for matching the two. Figure 4 shows the values of λ_N for



Figure 4. Sensitivity of mean path length for nuclei, λ_N , to parameters of the model. The full curves relate to the condition $DT \propto R^{\alpha}$ with $T \neq f(R)$ normalised to the peak of $f((DT)^{1/2})$ and the broken curves to $DT \propto R$ and $T \propto R^{-0.1}$ with the values of parameters given in § 3. The predictions are normalised to the experimental value at R = 5 GV.

the case where $T \neq f(R)$ and $DT \propto R^{\alpha}$ (full curves) normalising the peak of $f((DT)^{1/2})$ in figure 3 (A = 10) at 5 GV. A better fit arises if the normalisation is after the peak and T is allowed to vary with R. A combination which fits the data reasonably well (the broken curve of figure 4) has the following parameters:

$$(DT)^{1/2} = 1.8 \times 10^{22} \times R^{0.5}$$
 cm, with R in GV
 $T(R) = 9.9 \times 10^{14} R^{-0.1}$ s,

giving

$$D(R) = 3.2 \times 10^{29} R^{1.1} cm^2 s^{-1}$$

The form of D(R) is given in figure 2. (Note, we have now changed from E to the more rigorously precise rigidity, R).

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The T and D variations are chosen here first of all in order to fit the data on variation of λ_N with R. The expressions reproduce well the rather complicated form of the variation of grammage with particle rigidity assuming a simple power law variation for both D(R) and T(R). Strictly speaking the dependences given here are valid only in the rigidity interval 1-100 GeV/nucleon but they can probably be extrapolated some distance toward higher energies since the spectrum of the primary protons seems to be rather smooth up to 1000 GeV.

4. Cosmic ray gradient in the Galaxy

A consequence of the rapid fall-off with x of the production spectrum coupled with only moderate diffusion leads to a considerable gradient.

Figure 5 shows the results for the parameters adopted here.

It is necessary to see whether the gradient is consistent with the measured directional anisotropy and the observed γ -ray flux (which gives information about the particle flux at other points in the Galaxy).



Figure 5. Gradient of cosmic rays along the spiral arm. n(x)/n(50) represents the ratio of the cosmic ray intensity at distance x(kpc) from the Galactic centre to that locally.

The expected anisotropy is given by $\delta = (\lambda/n)(\partial n/\partial x)$. Inspection of figure 5 gives $(1/n)(\partial n/\partial x) \approx 0.07 R^{-0.33} \text{ kpc}^{-1}$ which, when combined with $\lambda = 10^{-2} R^{1.1}$ kpc from § 3, gives $\delta \approx 7 \times 10^{-4} R^{0.77}$ (with R in GV). At about 300 GV, where Marsden *et al* (1976) have made measurements which appear to indicate $\delta \approx 1.7 \times 10^{-3}$, the equation gives $\delta \approx 5 \times 10^{-2}$, a value considerably higher.

Another way of looking at the discrepancy is in terms of the mean free path for particle scatter, λ . For example, Osborne *et al* (1976), assuming that the anisotropy of Marsden *et al* is due to production of cosmic rays by the Vela supernova, came to the conclusion that the mean free path of particles of a few 100 GeV is of order 10 pc. The values obtained from the present work are about 1–2 kpc, i.e. at least one hundred times higher. As has been pointed out already, these high values of the mean free path are necessary if cosmic rays are to be collected from a volume of the order of the size of the Galaxy.

The apparent discrepancy can be understood if we assume that the diffusion coefficient is different in the galactic arm and in the halo, that is we modify the simple one-dimensional treatment adopted so far. This possibility has been widely discussed by Ginzburg and Ptuskin (1976) and the ratio $D_h/D_d \sim 100$ does not seem unreasonable.

The essence of the modified model is thus the following: cosmic rays are produced in sources distributed exponentially in the galactic arms. They diffuse into the halo where, due to the high diffusion coefficient, they propagate rapidly mainly along the galactic arm due to the alignment of the magnetic field. The cosmic rays from the halo also diffuse inside the galactic arms so that the locally observed particles of energy of the order of 100 GeV are predominantly those which were produced at large distances (several kiloparsecs) from us and propagated to us through the halo. The model probably also requires a reflecting boundary on the edge of the galactic halo and the energy-dependent transparency of the boundary can explain the obtained dependence of T on R.

It should be added that the model also explains the low intensity of 10 Be reported by Garcia-Munoz *et al* (1975).

Turning to γ rays, which so far give information in the range of proton energy 1-10 GeV, a very large radial gradient should be apparent. The analysis of γ -ray measurements towards the Galactic centre is rather unclear because of the uncertain amounts of molecular hydrogen in the inner Galaxy and the large increase indicated here probably cannot be ruled out (Wolfendale and Young 1977). Towards the anti-centre Dodds *et al* (1975) and Strong *et al* (1977) have indicated a gradient at least as steep as that of stellar density (number of stars per unit volume) and this would be quite consistent with the gradient considered here.

5. Relevance to flux of cosmic ray positrons

As pointed out in II a gradient of particles in the Galaxy, different for different energies, causes an inequality between λ_{e^+} and λ_N , as appears to be necessary in view of the analysis given in I.

The apparent grammage for positrons can be calculated from the expression

$$\lambda_{e^+} = \frac{\lambda_0 T_0}{(D_0 T_0)^{1/2}} \frac{1}{2\bar{t}_0} B \left[\exp\left(-\frac{|x|}{(D_1 T_1)^{1/2}}\right) - \frac{A}{(D_1 T_1)^{1/2}} \exp\left(-\frac{|x|}{A}\right) \right]^{-1}$$
(5.1)

where

$$B = \int_{-\infty}^{\infty} \left[\exp\left(-\frac{|x-u|}{(D_1T_1)^{1/2}}\right) - \frac{A}{(D_1T_1)^{1/2}} \exp\left(-\frac{|x-u|}{A}\right) \right] \exp\left(-\frac{|u|}{(D_0T_0)^{1/2}}\right) du.$$
(5.2)

Here λ_0 is the grammage of protons of the same rigidity and \overline{t}_0 their mean lifetime. The subscript '1' denotes parameters corresponding to the energy of the parent protons and '0' refers to the energy of the observed positrons.

Figure 6 shows the resulting value of λ_{e^+} compared with the summary of derived values given in I. Although the fit is not good it is tolerable.

6. Vacuum lifetime distribution of cosmic rays

The diffusion model adopted here can be used to predict the distribution of particle lifetimes as well as the mean value, \tilde{t} .



Figure 6. Comparison of the derived value of the apparent positron grammage, λ_{e^*} , with the experimental limits.

Figure 7 shows the distribution of lifetimes (or rather grammages) calculated for R = 5 GV. Comparison is made with the form derived by Shapiro and Silberberg (1974) from their analysis of the propagation of the various isotopic components. We consider the fit to be rather good.



Figure 7. Distribution of cosmic ray grammage P(l) from the model (for R = 5 GV) (curve A) and the distribution derived from analysis of experimental data of Shapiro and Silberberg (1974) (curve B).

7. Form of the production spectrum of nuclei

The relationship between the production spectrum and the locally measured spectrum $n_s(R)$ can be derived in a straightforward way from the model (using equation (2.2) and the forms for D(R) and T(R)). Writing $n_s(R) = q(R) \times F(R)$ the form of F(R) is that given in figure 8.

Clearly, above about 70 GeV the measured spectrum will be steeper than the production spectrum, the difference in exponents tending to 0.6 (since at high energies $n_s(R) \propto q(R) A(T/D)^{1/2}$). Thus a production spectrum of the form $q(R) \propto R^{-2.0}$ would appear to be necessary, with presumably a steepening at energies below some



Figure 8. Rigidity dependence of the ratio of the local cosmic ray spectrum, $n_s(R)$, to the production spectrum, q(R). The broken line is an extrapolation back of the high rigidity behaviour.

tens of GeV because $n_s(R)$ does not appear to be as flat as $R^{-2 \cdot 0}$ in this region, although there is in fact some reduction in the actual exponent of the measured spectrum below about 30 GeV.

The steeper spectrum in the low energy region can be expected if, to the flux of high energy cosmic rays originating in supernovae, there are added those from less powerful but more numerous sources such as novae, flare stars and perhaps even ordinary stars under flaring conditions.

Turning now to the question of energies above 10^3 GeV we expect that the adopted model will break down. The situation may well be that the proper diffusion will be replaced by some sort of movement governed by reflection from the boundaries of the halo. In that case the dependence of DT on E should become weaker and some flattening of the observed spectrum may be expected. This point needs further investigations both experimentally and theoretically.

8. Conclusions

We have used a simple one-dimensional diffusion model to draw attention to the fact that if cosmic rays below some hundreds of GeV (at least) are produced in the Galaxy then the resultant cosmic ray gradient not only allows an explanation of the positron data but also the variation of mean path length of nuclei with rigidity. The variation follows in a natural way if the diffusion coefficient and inherent lifetime of the particles are simple power laws of rigidity over the whole range.

Explanation of the measured low value of the anisotropy of particles in the region of several hundred GeV requires a modification of the model in the form of an assumed halo in which fast propagation is allowed but such an assumption appears reasonable.

The model indicates that the cosmic rays with energies above about 10 GeV are not of local origin, within say 1-2 kpc. This conclusion is in fact more general. It seems that in order to be able to explain the experimental intensity of positrons, significant differences in intensities of cosmic rays are needed in the Galaxy and this in turn requires large collection volumes. The present model is obviously only one of the possible models fulfilling this condition.

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